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THEORY OF PROPAGATION OF PARTIALLY COHERENT LIGHT BEAMS IN A TURBULENT ATMOSPHERE

A. I. Kon, et al

Army Foreign Science and Technology Center
Charlottesville, Virginia

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AUTHOR: A. I. Kon and V. I. Tatarskiy

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ABSTRACT: The function of mutual coherence is con-
sidered for a partially coherent light source
in a turbulent medium. The behaviour of the
effective radius of the coherence ρ_{eff} de-
termining the angular beamwidth is investigated.
The dependence of the mean light intensity in
the turbulent medium on the initial beam cohe-
rence is also considered. It is shown that
the mean intensities for coherent and incohe-
rent sources become close at a sufficiently
strong level of turbulence.

NOTICE

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This article considers the effects of turbulent fluctuations in dielectric permeability on the characteristics of a light beam which are described by its 2nd order coherence function. The majority of theoretical studies dealing with the propagation of light in homogeneous mediums consider purely coherent radiation sources. However the degree of coherence of the source can be an important factor in a number of cases even when we are interested in such comparatively coarse characteristic as the mean light intensity. Below we consider the sources of partially coherent radiation in a turbulent atmosphere and investigate the effect of the degree of coherence of the source on the characteristics of the light beam.

Let us consider the light source in the $x = 0$ plane. Its coherence function is

$$\Gamma^0(R, \rho) = \overline{u_0(R + (1/2)\rho)\, u_0^*(R - (1/2)\rho)}, \qquad (1)$$

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where u_0 is the field in the plane $x = 0$
and the bar denotes averaging of the field u_0
over the fluctuations. For a completely coherent source
Γ^0 the function has the form of a product,

$$\Gamma^0_{coh}(R, \rho) = u_0(R + (1/2)\rho)\, u_0^*(R - (1/2)\rho), \qquad (1a)$$

so that in this case the modulus of the complex
coherence [1] is equal to unity.

$$|\gamma(R, \rho)| = \frac{|\Gamma^0(R, \rho)|}{[|\Gamma^0(R+(1/2)\rho, 0)\, \Gamma^0(R-(1/2)\rho, 0)|]^{1/2}} = 1.$$

In a turbulent medium the coherence function is
$$\Gamma(x, R, \rho) = \langle u(x, R + (1/2)\rho)\, u^*(x, R - (1/2)\rho) \rangle,$$

where $u(x, R)$

is the field at the distance x from the source and
the brackets denote averaging over the fluctuations
in the dielectric permeability. It satisfies the
equation derived by Dolin [2] (See also [3-5]):

$$\frac{\partial \Gamma(x, R, \rho)}{\partial x} - \frac{i}{k} \nabla_R \nabla_\rho \Gamma + \frac{\pi k^2}{4} H(x, \rho) \Gamma = 0. \qquad (2)$$

Here $\quad H(x, \rho) = 2 \iint_{-\infty}^{\infty} \Phi_\epsilon(x, \varkappa)(1 - \cos \varkappa\rho)\, d^2\varkappa \qquad (3)$

and $\quad \Phi_\epsilon(x, \varkappa) = \Phi_\epsilon(x; 0, \varkappa_y, \varkappa_z)$ is the three-dimensional spectral
density of the fluctuations in the dielectric permea-
bility which is a smooth function of the longitudinal
coordinate x.

The solution of equation (2) with initial condition
(1a) has the form [3]

$$\Gamma(x, R, \rho) = \frac{k^2}{4\pi^2 x^2} \iint_{-\infty}^{\infty} d^2 R' \iint d^2\rho' \, \Gamma^0(R', \rho') \exp\left\{ \frac{ik}{x} (\rho - \rho') \times \right.$$

$$\left. \times (R - R') - \frac{\pi k^2}{4} \int_0^x H\left[x', \rho \frac{x'}{x} + \rho'\left(1 - \frac{x'}{x}\right) \right] dx' \right\}. \qquad (4)$$

When the function $\Gamma'(R', \rho') = u_0(R' + (1/2) \rho') u_0^*(R' - (1/2) \rho')$ which appears in (4) is considered as a random function and it is averaged over the fluctuations u_0, we obtain an expression for the function $\Gamma(x, R, \rho)$, corresponding to a partially coherent light source, with an initial coherence function in the general form (1).

We will now consider some consequences which follow from formula (4). We integrate (4) with respect to R and we introduce the notation

$$\gamma(x, \rho) = \iint_{-\infty}^{\infty} \Gamma(x, R, \rho) \, d^2R, \qquad \gamma_0(\rho) = \gamma(0, \rho). \qquad (5)$$

Then on the basis of (4), we obtain the simple relation

$$\gamma(x, \rho) = \gamma_0(\rho) \exp\left[-\frac{\pi k^2}{4} \int_0^x H(x', \rho) \, dx' \right] \qquad (6)$$

for the coherence function averaged over the beam [3].

The immediate physical meaning of $\gamma(x, \rho)$ is that it determines the angular spectrum of the light beam. In fact when we place a lens in the plane $x = $ const which intercepts the entire beam and consider the distribution of the intensity in its focal plane, the mean value of the intensity I at the distance r from the optical axis of the lens will be proportional to

$$I(r) \sim \iint \gamma(x, \rho) \exp\left(\frac{ikr}{f} \rho \right) d^2\rho. \qquad (7)$$

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where f is the focal length of the lens. Thus the characteristic scale of the function $\gamma(x,\rho)$ over ρ determines the angular beam width. We will call this characteristic scale the "effective radius of coherence."

It should be noted that the effective radius of coherence which was introduced in this way does not coincide with the radius of coherence which is defined as the characteristic scale of the complex degree of coherence γ. For example let us consider a pure coherent light source with a field distribution in its plane in the form

$$u_0(\rho) = A \exp\left(-\frac{\rho^2}{2a^2} - \frac{ik\rho^2}{2F}\right). \qquad (8)$$

Here a is the characteristic dimension of the beam, F is the radius of curvature of its phase front (when $F < 0$ the beam diverges). The complex degree of coherence corresponding to (8) is equal in modulus to unity since the radius of coherence of such a source is infinite. At the same time the function $\gamma_0(\rho)$, corresponding to (8) is equal to

$$\gamma_0(\rho) = \pi a^2 |A|^2 \exp\left[-\frac{1}{4}\left(\frac{1}{a^2} + \frac{k^2 a^2}{F^2}\right)\rho^2\right] \qquad (9)$$

and

$$\rho_{\text{eff}}^2 = \frac{4a^2 F^2}{k^2 a^4 + F^2}. \qquad (10)$$

When for example $|F| = \infty$, $\rho_{eff} = 2a$

even though the beam under consideration is completely
coherent. The infinite radius of coherence for a
source of type (8)
indicates in   principle the possibility of obtaining
for it a sharp interference pattern with $I_{min} = 0$.
When for example, one of the apertures of the interfero-
 meter is placed on the axis of the beam and the
other at a distance on the order a from it, to obtain
the interference pattern with $I_{min} = 0$
an attennuator is needed which equalizes the inten-
sities at both apertures. When such an attennuator
is not used, the inter ference pattern will spread
at distances on the order of ρ_{eff}.

Thus ρ_{eff} determines not only the angular
beam width but has also direct relevance on the pos-
sibility of obtaining an interference pattern when
amplitude-phase adjustors are not used.

Let us consider the case when the light source has
partial or three dimensional coherence. Suppose for
example

$$u_0(\rho_1) = A \exp\left[-\frac{\rho_1^2}{2a^2} - \frac{ik\rho_1^2}{2F} + i\varphi(\rho_1) \right], \qquad (8a)$$

where $\phi(\rho_1)$ is the random phase whose mean value is equal to
0 and has a Gaussian distribution. When

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$$\overline{[\varphi(\rho_1)\cdots\varphi(\rho_2)]^2} = F(\rho_1 - \rho_2),$$

we obtain easily for the function $\Gamma^0(R, \rho)$

the formula

$$\Gamma^0(R, \rho) = |A^2| \exp\left[-\frac{R^2}{a^2} - \frac{\rho^2}{4a^2} - \frac{ik\rho R}{F} - \frac{1}{2}F(\rho)\right].$$

In this case the function $\gamma_0(\rho)$

has the form

$$\gamma_0(\rho) = \pi a^2 |A^2| \exp\left[-\frac{1}{4}\left(\frac{1}{a^2} + \frac{k^2 a^2}{F^2}\right)\rho^2 - \frac{1}{2}F(\rho)\right].$$

Formula (9) was obtained taking into account the well known equality

$$\overline{\exp\{i[\varphi(\rho_1) - \varphi(\rho_2)]\}} = \exp\left\{-\frac{1}{2}\overline{[\varphi(\rho_1) - \varphi(\rho_2)]^2}\right\}.$$

In subsequent calculations we will use a quadratic approximation of the function $F(\rho)$

$$F(\rho) = \rho^2/2\rho_\kappa^2,$$

where ρ_κ is the radius of coherence of the source which is the characteristic scale of the complex degree of coherence $\gamma(R, \rho)$. We note that ρ_κ is related to the angular beam width ψ_0 in the region $x \gg x_0$, (where its directivity pattern was formed) by the expression

$$\psi_0 = \frac{1}{ka}\sqrt{1 + a^2/\rho_\kappa^2} \quad \left(x > x_0 = \frac{ka^2\rho_\kappa}{\sqrt{a^2 + \rho_\kappa^2}}\right),$$

which enables us to estimate ρ_κ.

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Then

$$\gamma_v(\rho) = \pi a^2 |A^2| \exp\left\{-\frac{1}{4}\left[\frac{1}{a^2} + \frac{k^2 a^2}{F^2} + \frac{1}{\rho_\kappa^2}\right]\rho^2\right\}. \qquad (9a)$$

Let us return to relation (6) and consider the
case of a statistically homogeneous turbulent medium
for which the structural dielectric permeability
function can be approximated by the expression

$$D_\epsilon(\varkappa, \rho) = C_\epsilon^2(x)\left[(\rho^2 + l_0^2)^{1/3} - l_0^{2/3}\right]. \qquad (11)$$

The spectral density

$$\Phi_\epsilon(\varkappa) = N C_\epsilon^2 l_0^{11/3}(\varkappa l_0)^{-11/6} K_{11/6}(\varkappa l_0),$$
$$N = [9 \cdot 2^{1/3} \pi^{3/2} \Gamma(5/3)]^{-1}. \qquad (12)$$

corresponds to structural function (11).

We note that the representation of the spectral
density in the form (12) approximates well the experi-
mental data [6] with respect to the temperature fluc-
tuations both in the inertial and in the viscous spec-
tral interval. On the other hand expression (11)
has a simple form and it is convenient to approximate
the structural function both in the inertial and vis-
cous interval. The scale l_0 is related to the
scale λ_{00} which is defined as the point of intersection
of the asymptotic expansions $D_\epsilon(\rho) \sim \rho^2$ and $D_\epsilon'(\rho) \sim \rho^{2/3}$,

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by the relation

$$r_0 l_0 = 3^{3/4}.$$

Substituting (12) in (3) after the integral is evaluated we obtain the formula

$$H(x, \rho) = MC_i^2(x)[(\rho^2 + l_0^2)^{5/6} - l_0^{5/3}],$$

$$M = \frac{4\Gamma(7/6)}{5\sqrt{\pi}\,\Gamma(5/3)} \approx 0.464. \qquad (13)$$

and in accordance with (9), (13) and (16) we find the coherence function averaged in the plane

$$\gamma(x, \rho) = \pi a^2 |A^2| \exp\left\{-\frac{1}{4}\left(\frac{1}{a^2} + \frac{1}{\rho_k^2} + \frac{k^2 a^2}{l^2}\right)\rho^2 - \right.$$

$$\left. - \frac{\pi M}{4}\,k^2 \overline{C_i^2}\,x\,[(\rho^2 + l_0^2)^{5/6} - l_0^{5/3}]\right\}, \qquad (14)$$

where we used the notation

$$\overline{C_i^2} = \frac{1}{x}\int_0^x C_i^2(x')\,dx'.$$

Formula (14) enables us to investigate the behavior of the quantity ρ_{eff} in a turbulent medium. When we define ρ_{eff} as that value of ρ, for which $\gamma(x, \rho)$ decreases e times in comparison $\gamma(x, 0)$, we obtain for ρ_{eff} the equation

$$\left(\frac{1}{a^2} + \frac{k^2 a^2}{l^2} + \frac{1}{\rho_k^2}\right)\rho_{eff}^2 + \pi M \overline{C_i^2}\,k^2 x\,[(\rho_{eff}^2 + l_0^2)^{5/6} - l_0^{5/3}] = 4, \qquad (15)$$

whose solution has the form

$$\rho_{eff}^2 = l_0^2\left[\left(\frac{B}{\alpha}\right)^6 \zeta(\eta) - 1\right]. \qquad (16)$$

Here

$$B = \frac{1}{4}\pi M \overline{C_i^2} x l_0^{-1/3} k^2, \qquad \alpha = \frac{1}{4} l_0^2 \left(\frac{1}{a^2} + \frac{k^2 a^2}{F^2} + \frac{1}{\rho_k^2} \right),$$

$$\gamma = \left(\frac{\alpha}{B} \right)^5 \left(1 + \frac{\alpha + 1}{B} \right)$$

and $\zeta(\gamma)$ is a root of the equation

$$\zeta + \zeta^{5,6} = \gamma. \tag{17}$$

In the region $\gamma \ll 1$, which corresponds to $\alpha \ll B$, the function $\zeta(\gamma)$ is

$$\zeta(\gamma) \approx \gamma^{6/5}.$$

In this case

$$\rho_{eff}^2 \approx l_0^2 \left[\left(1 + \frac{1}{B} \right)^{6/5} - 1 \right], \qquad B \gg \alpha. \tag{18}$$

When the conditions $B \gg \alpha$, $B \gg 1$, are satisfied simultaneously, we obtain from (18)

$$\rho_{eff}^2 = \frac{6}{5} \frac{l_0^2}{B}, \qquad B \gg \alpha, \quad B \gg 1, \tag{18a}$$

and the effective radius of coherence is small in comparison with the internal turbulence scale l_0.

When $B \gg \alpha$, but $B \ll 1$, i.e. $\alpha \ll B \ll 1$, we obtain from (18)

$$\rho_{eff}^2 = \left(\frac{4}{\pi M \overline{C_i^2} x} \right)^{6/5}, \qquad 1 \gg B \gg \alpha. \tag{18b}$$

In both cases (18a) and (18b) the radius of coherence is determined only by the turbulence parameters and it does not depend on the initial parameters of the beam. Therefore we can call the case $B \gg \alpha$

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the case of strong turbulence.

For the condition $\gamma \gg 1$, which can also hold when $B \ll a$ or $a \approx B \ll 1$, the solution of equation (7) takes on the form

$$\zeta(\gamma) = \gamma \left[1 - \gamma^{-1/6} + \frac{5}{6}\, \gamma^{-2/6} - \frac{55}{72}\, \gamma^{-3/6} + \dots \right],$$

and we obtain instead of (16)

$$\rho_{\psi\psi}^2 = \frac{4}{1\,a^2 + k^2 a^4 F^2 + 1\,\rho_k^2} \left\{ 1 - B\left[\left(1 + \frac{1}{\alpha} \right)^{5/6} - 1 \right] + O(B^2) \right\}. \quad (19)$$

The last formula applies to the case of weak turbulence when the effective radius of coherence is mainly determined by the light source.

Let us now consider the mean intensity which can be obtained from (4), by setting $\rho = 0$. In the previous model of the light source after the quadratic approximation of the function $F(\rho) = \rho^2/\rho_k^2$ is substituted in (9), the expression for $I(x, R) = I(x, R, 0)$ can be reduced to the form

$$I(x, R) = \frac{1}{2} \left(\frac{ka|A|}{x} \right)^2 \int_0^\infty J_0\left(\frac{kR\rho'}{x} \right) \times$$

$$\times \exp\left[-\frac{g_1^2(x)}{4a^2} \rho'^2 - \frac{\pi k^2}{4} \int_0^x II(x', \rho'(1-x'/x))\, dx' \right] \rho'\, d\rho' \quad (20)$$

after integration with respect to R' and the angular variable related to ρ'. Here we used the notation

$$g_1(x) = \left[1 + k^2 a^4 \left(\frac{1}{x} - \frac{1}{F} \right)^2 + \frac{a^2}{\rho_k^2} \right]^{1/2}. \quad (21)$$

We note that the radius of coherence of the source ρ_k enters (20) only by way of the parameter $g_1(x)$.

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When we consider a pure coherent light source for
which $g = g$, instead of $g_1(x)$ we obtain the function

$$F(x) = \left[1 + k^2 a^4 \left(\frac{1}{x} - \frac{1}{F} \right)^2 \right]^{1/2}, \qquad (21a)$$

and the expressions (20), (21) coincide with those
given in [3]. Thus the incomplete coherence of this
source in the expression for the mean intensity is equi-
valent to some change in the initial radius of curvature F.

Let us consider in greater detail the expression
for the mean intensity on the beam axis. We use the
same model (13) for the function $H(x, \rho)$, and we
assume $C_1(x) =$ constant. Then we obtain
easily for $\bar{I}(x, 0)$ the formula

$$\bar{I}(x, 0) = I_0(x) \, p \cdot \int_0^\infty \exp \left\{ -pt - B \int_0^1 [(t\,\xi^2 + 1)^{5/6} - 1] \, d\xi \right\} dt, \quad (22)$$

where

$$p = \left(\frac{L_0 g_1(x)}{2a} \right)^2, \quad I_0(x) = \left(\frac{ka^2 A}{x g_1(x)} \right)^2,$$

and B is defined by (16). The quantity $I_0(x)$ is the
intensity on the beam axis from the same light source
in the absence of turbulence.

In the case of small p, expression (22) takes on
the asymptotic form

$$\bar{I}(x, 0) = I_0(x) \int_0^\infty \exp(-t - \mu t^{5/6}) \, dt = I_0(x) f(\mu), \quad (23)$$

$$Bp^{5/6} \ll 1.$$

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where $\mu = \frac{3}{8} Bp^{-5/6}$. We note that expression (23) is easily obtained directly from (20) also in the case of variable C_i.

For the condition $Bp^{1/6} \ll 1$ the asymptote $H(x, \rho) = MC_\epsilon^2(x)\rho^{5/3}$, which after substitution in (20), leads to expression (23) in which

$$\mu = \frac{\pi M}{4} k^2 x \left(\frac{2a}{g_1(x)} \right)^{5/3} \int_0^1 C_i^2(\eta x)(1-\eta)^{5/3} d\eta. \qquad (24)$$

The function $f(\mu)$ was studied in [3]. Here

$$\mu = \frac{1}{2} D_{1,sph}\left(x, \frac{2a}{g_1(x)} \right),$$

where $D_{1,sph}$ is the root mean square fluctuation of the complex phase for a spherical wave. Thus $\bar{I}$ depends on the root mean square phase difference on the base $2a/g_1(x)$. When we consider a weak coherent source for which $\rho_k \ll a$, $\rho_k \lambda x/a$, $g_1(x) \approx \frac{a}{\rho_k}$ and $\frac{2a}{g_1} \approx 2\rho_k$. In this case the intensity depends on the root mean square phase difference on the radius of coherence.

The ratio of the mean intensity on the beam axis for a partially and completely coherent source is given in Fig. 1 as a function of ρ_k/a. The different curves differ by the parameter μ_{coh} which is determined from expression (24), in which

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g_1 was replaced by g. When both inequalities $\mu_{coh} \gg 1$, $\mu \gg 1$, are satisfied, in both cases the asymptotic formula [3] $f(\mu) = 1.10\mu^{-6/5} - 1.49\mu^{-12/5} + \dots$ holds, and

$$\frac{I_{incoh}}{I_{coh}} \approx 1 - 1.35\left(\frac{1}{\mu^{6/5}} - \frac{1}{\mu_{coh}^{6/5}}\right) + \dots$$

and the quantities $\bar{I}_{coh}$ and $\bar{I}_{incoh}$ are close to one another, i.e., in the region $\mu \gg 1$ the mean intensity no longer depends on the degree of coherence of the source.

Fig. 1. Ratio of Mean Intensities on the Beam Axis For a Partially Coherent And Completely Coherent Source (Curve 1 when there is no turbulence, curve 2, $\mu_{coh}=3$, curve 3, $\mu_{coh}=10$
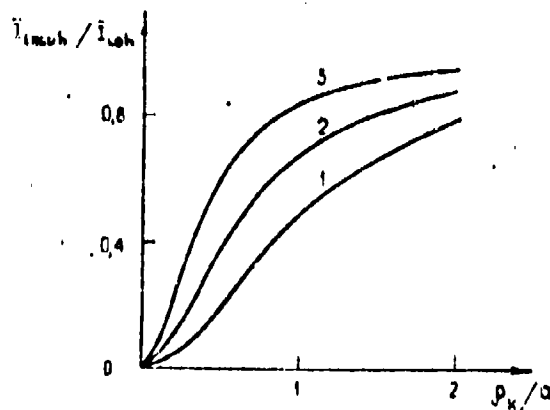
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